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On the Orbit of the Binary Star ξ Ursæ Majoris.[*]
By Dr. Robert S. Ball.

The object of this paper is to compare some elements which have been computed, with recent observations; to propose a new set of approximate elements; and to give an approximate ephemeris.

The last periastre occurred in 1816·4. Prior to this date the only observations are those of Herschel I. in 1781, 1802, 1804. In the interval of fifteen years from 1804 to 1819 the position-angle swept through 168°, yet no observation has been recorded during this critical period.[†] The star will reach periastre again in 1876·3, consequently recent observations have to a great extent replaced those wanting between 1804 and 1819.

The four sets of elements here examined are taken from Herschel's *Outlines*, 4th edition, p. 573. Savary's value of the inclination is 59° 40′. By a misprint it is given 50° 40′ in the *Outlines*.

Table I. *ξ Ursæ Majoris.*

Eccentricity. e	Position of Node. Ω	Inclination. γ	Angle between Major Axis and Nodes. λ	Period in Years. P	Perihelion Passage A.D. ε	Authority.
	° ′	° ′	° ′			
0·41640	95 22	59 40	131 38	58·262	1817·25	Savary.
0·37770	97 47	56 6	134 22	60·720	1816·73	Herschel II.
0·41350	98 52	54 56	130 48	61·464	1816·44	Mädler.
0·43148	95 50	52 49	128 57	61·576	1816·86	Villarceaux.

In the comparison of these elements with observation, twenty-two observations have been selected.[‡] These observations are distributed as well as possible over the ninety years during which the star has been observed; and whenever a choice has been possible, results of the greatest weight have been taken. The result of the comparison is shown in Table II. (See next page.)

It will be noticed that Savary's elements represent the observations up to No. 6 very well. After No. 6 the calculated position falls behind the observed; the difference reaches a maximum in No. 13, where it amounts to —9°·44. Savary's elements, though the earliest in date, represent the orbit fairly.

The elements given by Herschel II. give for No. 22 a calculated position 17°·43 in excess of the observed position: with this exception, the greatest difference is — 6°·06 in No. 13. With reference to this determination, Sir John Herschel remarks, "The case is one highly unfavourable for the application of my method, and, moreover, the resulting elements are those of a first approximation only."

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[*] Read before the Royal Irish Academy.

[†] At least no such observation is recorded in Mr. Dawes' invaluable list, *M. R. A. S.* vol. xxxv. p. 353.

[‡] Many of these observations have been taken from Dawes' list. Others have been supplied by the kindness of Dr. Brünnow.

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Table II. ξ *Ursæ Majoris.*

				Savary.		Herschel II.		Mädler.		Villarceaux.	
No.	Observer.	Epoch.	Observed Position.	Calculated Position.	Diff. from Observed Position.	Calculated Position.	Diff. from Observed Position.	Calculated Position.	Diff. from Observed Position.	Calculated Position.	Diff. from Observed Position.
			°	°	°	°	°	°	°	°	°
1	Herschel I. ..	1781·97	143·78	143·89	+0·11	140·08	− 3·70	142·85	− 0·93	144·12	+ 0·34
2	„ ..	1802·09	97·52	97·40	−0·12	93·92	− 3·60	97·72	+ 0·20	97·73	+ 0·21
3	„ ..	1804·08	92·63	93·07	−0·34	88·52	− 4·11	92·45	− 0·18	92·55	− 0·08
4	Struve ..	1819·10	284·55	284·97	+0·42	285·13	+ 0·58	284·54	− 0·10	285·60	+ 0·05
5	Herschel II. and South	1823·29	258·45	257·45	−1·00	258·37	− 0·08	256·60	− 1·85	254·78	− 3·67
6	South ..	1825·22	244·53	245·50	−0·97	246·45	+ 1·92	244·47	− 0·06	242·38	− 2·15
7	Struve ..	1827·27	228·27	231·50	+3·23	232·68	+ 4·41	230·97	+ 2·70	229·03	− 0·76
8	Dawes ..	1832·27	196·72	191·69	−5·03	195·40	− 1·32	196·39	− 0·33	196·43	− 0·29
9	„ ..	1840·29	150·85	143·65	−7·20	148·92	− 1·93	153·80	+ 2·95	155·48	+ 4·63
10	„ ..	1842·27	144·76	136·50	−8·26	141·48	− 2·28	146·60	+ 1·84	148·33	+ 3·57
11	„ ..	1843·28	142·17	133·29	−8·88	138·15	− 4·02	143·35	+ 1·18	144·98	+ 2·81
12	„ ..	1849·30	126·58	118·44	−8·14	122·27	− 4·31	127·47	+ 0·89	128·58	+ 2·00
13	Miller ..	1852·13	122·28	112·84	−9·44(!)	116·22	− 6·06	121·37	− 0·91	122·22	− 0·06
14	Jacob ..	1853·20	119·47	110·85	−8·62	114·03	− 5·44	119·17	− 0·31	119·93	+ 0·46
15	Dawes ..	1854·36	115·87	108·72	−7·15	111·73	− 4·14	116·85	+ 0·98	117·55	+ 1·68
16	Dembowski ..	1858·20	108·09	101·67	−6·42	104·12	− 3·97	109·29	+ 1·20	109·70	+ 1·61
17	„ ..	1863·23	96·66	90·92	−5·74	92·85	− 3·81	98·54	+ 1·88	98·48	+ 1·82
18	..	1865·12	91·42	85·59	−5·83	87·55	− 3·87	93·67	+ 2·25	94·00	+ 2·58
19	Dembowski ..	1866·30	86·76	81·50	−5·26	83·65	− 3·11	90·20	+ 3·46	90·68	+ 3·92
20	„ ..	1867·31	82·22	77·24	−4·98	79·78	− 2·44	86·82	+ 4·60	87·52	+ 5·30
21	„ ..	1868·30	77·50	72·02	−5·48	75·27	− 2·23	83·02	+ 5·52	84·02	+ 6·52
22	Brünnow ..	1872·28	24·19	20·57	−3·62	41·62	+17·43(!)	57·19	+33·00(!)	62·10	+37·91(!)

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Mädler's elements represent the first eighteen observations with surprising fidelity, and there is no very serious discrepancy until No. 22, when the difference is 33°.

Villarceaux's elements give for No. 22 a discrepancy of 37°·91.

There cannot be a doubt that, had the four eminent astronomers who have been mentioned had the advantage of several observations in the neighbourhood of periastre, their orbits would have been verified by the recent observations. Three of the orbits, Savary's being the exception, appear to have too large a value for the periodic time. This is the principal source of the discrepancies. In the absence of Mr. Brünnow's observation No. 22, and some others preceding it, Mädler's elements would have fair claims, but it will be evident that some change is rendered necessary by the recent observations.

By the aid of Sir John Herschel's beautiful graphic process, *M. R. A. S.* (vol. v. p. 209), the following approximate elements have been determined:—

Ω = 103°·6
γ = 53 ·1
λ = 135 ·3
P = 59·88
ε = 1816·405

These elements have been compared with the twenty-two observations already mentioned, and also with two more, Nos. 22 and 23 of the following Table:—

Table III. ξ *Ursæ Majoris.*

Comparison of the new Approximate Elements,

No.	Authority.	Epoch.	θ_c the Observed Angle of Position.	θ_c the Computed Angle of Position.	Difference $\theta_c - \theta_c$
			°	°	°
1	Herschel I. ..	1781·97	143·78	147·37	+3·59
2	,, ..	1802·09	97·52	98·53	+1·01
3	,, ..	1804·08	92·63	92·55	—0·08
4	Struve ..	1819·10	284·55	287·93	+3·38
5	Herschel II. and South	1823·29	258·45	259·20	+0·75
6	South ..	1825·22	244·53	246·20	+1·67
7	Struve ..	1827·27	228·27	231·63	+3·36
8	Dawes ..	1832·27	196·72	195·13	−1·59
9	,, ..	1840·29	150·85	152·95	+1·10
10	,, ..	1842·27	144·76	145·98	+1·22
11	,, ..	1843·28	142·17	142·85	+0·68
12	,, ..	1849·30	126·58	127·10	+0·52
13	Miller ..	1852·13	122·28	120·87	−1·41

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No.	Authority.		Epoch.	θ_c the Observed Angle of Position. °	θ_c the Computed Angle of Position. °	Difference. $\theta_c - \theta_c$ °
14	Jacob	..	1853·20	119·47	118·58	−0·89
15	Dawes	..	1854·36	115·87	116·12	+0·25
16	Dembowski	..	1858·20	108·09	107·80	−0·29
17	,,	..	1863·23	96·66	94·87	−1·79
18		..	1865·12	91·42	88·43	−2·99
19	Dembowski	..	1866·30	86·76	83·52	−3·24
20	,,	..	1867·31	82·22	78·55	−3·67
21	,,	..	1868·30	77·50	72·58	−4·92(!)
22	,,	..	1870·24	57·74	56·10	−1·64
23	,,	..	1871·22	47·70	43·80	−3·90
24	Brünnow	..	1872·28	24·19	26·47	+2·28

The greatest discrepancy is in No. 21, where the difference amounts to — 4°·92.

The angle of position is at present changing at the rate of nearly 20° per annum, consequently observations during the next few years will be of the greatest service in sidereal dynamics.

From the approximate elements which have been given the following approximate ephemeris has been computed A comparison of the positions given in this ephemeris with observations may perhaps provide materials for the final correction of the elements. The angle of position is computed at intervals of three months from 1872·50 up to 1878·75. The star will pass periastre in 1876·28.

Table IV. Approximate Ephemeris of the Angle of Position of *ξ Ursæ Majoris.*

No.	Epoch.	Angle of Position. °	No.	Epoch.	Angle of Position. °
1	1872·50	22·4	14	1875·75	321·4
2	·75	17·5	15	1876·00	317·9
3	1873·00	12·5	16	·25	314·7
4	·25	7·3	17	·50	311·6
5	·50	2·2	18	·75	308·6
6	·75	357·2	19	1877·00	305·8
7	1874·00	352·1	20	·25	303·2
8	·25	347·0	21	·50	300·7
9	·50	342·2	22	·75	298·4
10	·75	337·6	23	1878·00	296·1
11	1875·00	333·2	24	·25	293·9
12	·25	329·0	25	·50	291·8
13	·50	325·1	26	·75	289·7

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Essay on the Arc of the Meridian measured in Lapland. By I. Todhunter, M.A., F.R.S. *Transactions of the Cambridge Philosophical Society*, vol. xii. part I.

In this essay, Mr. Todhunter gives a series of valuable and interesting critical remarks on the accounts which have been given of the two measurements of the celebrated Lapland arc. He points out many (and some most remarkable) errors in these accounts, and applies the appropriate corrections.

It would be altogether impossible to give an abstract of the essay, which is already singularly condensed. Every line should be carefully studied by those who desire to know the real history of the important operations in question.

The following reflections are instructive and suggestive:—

"It would be a curious subject of speculation whether the theoretical opinions of persons engaged in geodetical surveys could have exercised any influence on their observations; I mean of course unconsciously, for it would be wrong to suspect any deliberate unfairness in any of the operations which I have examined. From a passage in the article *Figure de la Terre* by D'Alembert, in the original *Encyclopédie* it would appear that the school of Cassini originally believed that in consequence of the oblate form of the Earth, the length of a degree of the meridian *would decrease* from the equator to the pole. It seems strange, perhaps, now to suppose that such an error could be seriously maintained; but there can be no doubt of it: for example, the error was vehemently maintained by Keill, a man of some reputation, who was ultimately a Savilian professor at Oxford. See Keill's *Examination of Dr. Burnet's Theory of the Earth,* p. 140. It is certainly a remarkable coincidence that the school of Cassini, starting with the erroneous theoretical notion that the degrees of the meridian *ought* to decrease from the equator to the pole, arrived at the same result by observation and measurement. There can, I think, be no doubt that at least Maupertuis and Clairaut, who were the most eminent of the French party, held the correct Newtonian theory as to the figure of the Earth; and their result was rather too decided in its confirmation of this theory. Now the geodetical angles could scarcely be influenced by the theoretical opinions of the observers; because it would not be obvious in what way the result would be affected by an error in an angle. But in measuring the base it would of course be obvious that the larger was the value obtained, the stronger was the evidence for an oblate form. Similarly in estimating the amplitude, the smaller the value obtained the stronger was the evidence for the oblate form. In these two parts of the survey then it would be necessary to be on the watch lest the conviction of what the result ought to be should influence the impression of what the observation really gives. It is curious that Maupertuis and his party seem to have thought at first that

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their success was too decided and therefore their amplitude too small; and that on their second determination they should have made it between 3″ and 4″ larger than at first. Svanberg, I consider, was sent to Lapland with a strong expectation that he would obtain a less value of a degree of the meridian than that of Maupertuis. See Melanderhjelm's preface to Svanberg's book."

Mr. Todhunter makes the following remarks on a question of mathematical history:—

"Svanberg alludes to the researches of Huygens on centrifugal force; and then he says that if we suppose with Huygens that the force of gravity resides in the centre of the Earth we obtain for the ellipticity $\frac{1}{578\cdot73}$. Then he proceeds with the strange statement that these discoveries were the precursors of the hypothesis which Newton afterwards proposed of universal gravitation. Thus Svanberg ascribes to Huygens and not to Newton the priority in investigations of the figure of the Earth. Now the first edition of the *Principia* was published in 1686. Huygens published in 1690 his *Discours de la cause de la Pesanteur.* Although it appears from the preface that some part of this work had been long before communicated to the Academy at Paris, yet Huygens excepts the part which relates to the motion of pendulums as affected by the figure of the Earth: and all that relates to the calculation of the figure of the Earth he expressly says was added after the publication of Newton's *Principia.* Perhaps Svanberg was misled by a hasty glance at Bailly's *Histoire de l'Astronomie Moderne,* vol. iii. p. 9. Bailly accidentally notices Huygens before Newton. Again Svanberg says, 'Newton ne tarda pas à remarquer, que l'hypothèse de Huygens d'une force unique tendante au centre de la terre, étoit équivalente à celle d'y supposer une densité infinie.' I do not know what this means. Huygens really used the hypothesis of a force constant in magnitude, and directed always to the centre of the Earth; it is of course only as to the latter part of the hypothesis that we can assert the equivalence of an infinite density at the centre. I do not think that Newton drew attention to this equivalence: Clairaut did in the *Philosophical Transactions,* vol. xl. pp. 277–306. Clairaut, I think, was the first who showed that Huygens' value of the ellipticity follows approximately from the latter part of Huygens' hypothesis, namely, that the force always passes through the centre, without assuming any particular law of intensity of the force at different distances. See Clairaut's *Figure de la Terre,* p. 141. See also Plana, *Astronomische Nachrichten,* No. 839."

The Orbit of Castor. By J. M. Wilson, Esq.

In the last number of the *Monthly Notices* I stated that the result of an examination of the orbit of *Castor* by graphical